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# Real Interest Rate Persistence: Evidence and Implications

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## Abstract

The real interest rate plays a central role in many important financial and macroeconomic models, including the consumption-based asset pricing model, neoclassical growth model, and models of the monetary transmission mechanism. We selectively survey the empirical literature that examines the time-series properties of real interest rates. A key stylized fact is that postwar real interest rates exhibit substantial persistence, shown by extended periods of time where the real interest rate is substantially above or below the sample mean. The finding of persistence in real interest rates is pervasive, appearing in a variety of guises in the literature. We discuss the implications of persistence for theoretical models, illustrate existing findings with updated data, and highlight areas for future research.

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## 1. Introduction

The real interest rate—an interest rate adjusted for either realized or expected inflation—is the relative price of consuming now rather than later.<sup>1</sup> As such, it is a key variable in important theoretical models in finance and macroeconomics, such as the consumption-based asset pricing model (Lucas, 1978; Breeden, 1979; Hansen and Singleton, 1982, 1983), neoclassical growth model (Cass, 1965; Koopmans, 1965), models of central bank policy (Taylor, 1993), and numerous models of the monetary transmission mechanism.

The theoretical importance of the real interest rate has generated a sizable literature that examines its long-run properties. This paper selectively reviews this literature, highlights its central findings, and analyzes their implications for theory. We illustrate our study with new empirical results based on U.S. data. Two themes emerge from our review: (1) Real rates are very persistent, much more so than consumption growth; and (2) researchers should seriously explore the causes of this persistence.

First, empirical studies find that real interest rates exhibit substantial *persistence*, shown by extended periods of time where postwar real interest rates are substantially above or below the sample mean. Researchers characterize this feature of the data with several types of models. One group of studies uses unit root and cointegration tests to analyze whether shocks permanently affect the real interest rate—that is, whether the real rate behaves like a random walk. Such studies often report evidence of unit roots, or—at a minimum—substantial persistence. Other studies extend standard unit root and cointegration tests by considering whether real interest rates are fractionally integrated or exhibit significant nonlinear behavior,

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<sup>1</sup> Heterogeneous agents face different real interest rates, depending on horizon, credit risk, and other factors. And inflation rates are not unique, of course. For ease of exposition, this paper ignores such differences as being irrelevant to the economic inference.

such as threshold dynamics or nonlinear cointegration. Fractional integration tests typically indicate that real interest rates revert to their mean very slowly. Similarly, studies that find evidence of nonlinear behavior in real interest rates identify regimes where the real rate behaves like a unit root process. Another important group of studies reports evidence of structural breaks in the means of real interest rates. Allowing for such breaks reduces the persistence of deviations from the regime-specific means, so breaks reduce local persistence. The structural breaks themselves, however, still produce substantial global persistence in real interest rates.

The empirical literature thus finds that persistence is pervasive. While researchers have used sundry approaches to model persistence, certain approaches are likely to be more useful than others. Comprehensive model selection exercises are thus an important area for future research, as they will illuminate the exact nature of real interest rate persistence.

The second theme of our survey is that the literature has not adequately addressed the *economic causes* of persistence in real interest rates. Understanding such processes is crucial for assessing the relevance of different theoretical models. We discuss potential sources of persistence and argue that monetary shocks contribute to persistent fluctuations in real interest rates. While identifying economic structure is always challenging, exploring the underlying causes of real interest rate persistence is an especially important area for future research.

The rest of the paper is organized as follows. The next section reviews the predictions of economic and financial models for the long-run behavior of the real interest rate. This informs our discussion of the theoretical implications of the empirical literature's results. After distinguishing between *ex ante* and *ex post* measures of the real interest rate, the third section reviews papers that apply unit root, cointegration, fractional integration, and nonlinearity tests to real interest rates. The fourth section discusses studies of regime switching and structural breaks

in real interest rates. The fifth section considers sources of the persistence in the U.S. real interest rate and ultimately argues that it is a monetary phenomenon. The sixth section concludes.

## 2. Theoretical Background

### 2.1. Consumption-Based Asset Pricing Model

The canonical consumption-based asset pricing model of Lucas (1978), Breeden (1979), and Hansen and Singleton (1982, 1983) posits a representative household that chooses a real consumption sequence,  $\{c_t\}_{t=0}^{\infty}$ , to maximize  $\sum_{t=0}^{\infty} \beta^t u(c_t)$ , subject to an intertemporal budget constraint, where  $\beta$  is a discount factor and  $u(c_t)$  is an instantaneous utility function. The first-order condition leads to the familiar intertemporal Euler equation,

$$E_t \{ \beta [u'(c_{t+1}) / u'(c_t)] (1 + r_t) \} = 1, \quad (1)$$

where  $1 + r_t$  is the gross one-period real interest rate (with payoff at period  $t + 1$ ) and  $E_t$  is the conditional expectation operator. Researchers often assume that the utility function is of the constant relative risk aversion form,  $u(c_t) = c_t^{1-\alpha} / (1-\alpha)$ , where  $\alpha$  is the coefficient of relative risk aversion. Combining this with the assumption of joint log-normality of consumption growth and the real interest rate implies the log-linear version of the first-order condition given by (1) (Hansen and Singleton, 1982, 1983):

$$\kappa - \alpha E_t [\Delta \log(c_{t+1})] + E_t [\log(1 + r_t)] = 0, \quad (2)$$

where  $\Delta \log(c_{t+1}) = \log(c_{t+1}) - \log(c_t)$ ,  $\kappa = \log(\beta) + 0.5\sigma^2$ , and  $\sigma^2$  is the constant conditional variance of  $\log[\beta(c_{t+1}/c_t)^{-\alpha}(1 + r_t)]$ .

Equation (2) links the conditional expectations of the growth rate of real per capita consumption  $[\Delta \log(c_{t+1})]$  with the (net) real interest rate  $[\log(1+r_t) \cong r_t]$ . Rose (1988) argues that if (2) is to hold, then these two series must have similar integration properties. While  $\Delta \log(c_{t+1})$  is almost surely a stationary process  $[\Delta \log(c_{t+1}) \sim I(0)]$ , Rose (1988) presents evidence that the real interest rate contains a unit root  $[r_t \sim I(1)]$  in many industrialized countries. A unit root in the real interest rate combined with stationary consumption growth means that there will be permanent changes in the level of the real rate not matched by such changes in consumption growth, so (2) apparently cannot hold.

Figure 1 illustrates the problem identified by Rose (1988) using U.S. data for the *ex post* three-month real interest rate and annualized growth rate of per capita consumption (nondurable goods plus services) for 1953:1–2007:2. While the two series appear to track each other reasonably well for long periods, such as the 1950s, 1960s, and 1984–2001, they also diverge for significant periods, such as the 1970s, early 1980s, and 2001–2005.

The simplest versions of the consumption-based asset pricing model are based on an endowment economy with a representative household and constant preferences. The next subsection discusses the fact that more elaborate theoretical models allow for some changes in the economy—for example, changes in fiscal or monetary policy—to alter the steady-state real interest rate while leaving steady-state consumption growth unchanged. That is, they permit a mismatch in the integration properties of the real interest rate and consumption growth.

## 2.2. Equilibrium Growth Models and the Steady-State Real Interest Rate

General equilibrium growth models with a production technology imply Euler equations similar to (1) and (2) that suggest sources of a unit root in real interest rates. Specifically, the Cass (1965) and Koopmans (1965) neoclassical growth model with a representative profit-maximizing firm and utility-maximizing household predicts that the steady-state real interest rate is a function of time preference, risk aversion, and the steady-state growth rate of technological change (Blanchard and Fischer, 1989, Chap. 2; Barro and Sala-i-Martin, 2003, Chap. 3; Romer, 2006, Chap. 2). In this model the assumption of constant relative risk aversion utility implies the following familiar steady-state condition:

$$r^* = \zeta + \alpha z, \quad (3)$$

where  $r^*$  is the steady-state real interest rate,  $\zeta = -\log(\beta)$  is the rate of time preference, and  $z$  is the (expected) steady-state growth rate of labor-augmenting technological change. Equation (3) implies that a permanent change in the exogenous rate of time preference, risk aversion, or long-run growth rate of technology will affect the steady-state real interest rate.<sup>2</sup> If there is no uncertainty, the neoclassical growth model implies the following steady-state version of the Euler equation given by (2):

$$-\zeta - \alpha [\Delta \log(c)]^* + r^* = 0, \quad (4)$$

where  $[\Delta \log(c)]^*$  represents the steady-state growth rate of  $c_t$ . Substituting the right-hand side of (3) into (4) for  $r^*$ , one finds that steady-state technology growth determines steady-state consumption growth:  $[\Delta \log(c)]^* = z$ .

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<sup>2</sup> Changes in distortionary tax rates could also affect  $r^*$  (Blanchard and Fischer, 1989, pp. 56–59).

If the rate of time preference ( $\zeta$ ), risk aversion ( $\alpha$ ), and/or steady-state rate of technology growth ( $z$ ) change, then (3) requires corresponding changes in the steady-state real interest rate. Depending on the size and frequency of such changes, real interest rates might be very persistent, exhibiting unit root behavior and/or structural breaks. Of these three factors, a change in the steady-state growth rate of technology—such as those that might be associated with the “productivity slowdown” of the early 1970s and/or the “New Economy” resurgence of the mid-1990s—is the only one that will alter both the real interest rate and consumption growth, producing nonstationary behavior in both variables. Thus, it cannot explain the mismatch in the integration properties of the real interest rate and consumption growth identified by Rose (1988).

On the other hand, shocks to the preference parameters  $\zeta$  and  $\alpha$  will only change the steady-state real interest rate and not steady-state consumption growth. Therefore, changes in preferences potentially disconnect the integration properties of real interest rates and consumption growth. Researchers generally view preferences as stable, however, making it unpalatable to ascribe the persistence mismatch to such changes.<sup>3</sup>

In more elaborate models, still other factors can change the steady-state real interest rate. For example, permanent changes in government purchases and their financing can also affect the steady-state real rate in overlapping generations models with heterogeneous households (Samuelson, 1958; Diamond, 1965; Blanchard, 1985; Blanchard and Fischer, 1989, Chap. 3; Romer, 2006, Chap. 2). Such shocks affect the steady-state real interest rate without affecting steady-state consumption growth, so they potentially explain the mismatch in the integration properties of the real interest rate and consumption growth examined by Rose (1988).

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<sup>3</sup> Some researchers appear more willing to allow for changes in preferences over extended period. For example, Clark (2007) argues that a steady decrease in the rate of time preference is responsible for the downward trend in real interest rates in Europe from the early medieval period to the eve of the Industrial Revolution.



Finally, some monetary growth models allow for changes in steady-state money growth to affect the steady-state real interest rate. The seminal models of Mundell (1963) and Tobin (1965) predict that an increase in steady-state money growth lowers the steady-state real interest rate, and more recent micro-founded monetary models have similar implications (Weiss, 1980; Espinosa-Vega and Russell, 1998a,b; Bullard and Russell, 2004; Reis, 2007; Lioui and Poncet, 2008). Again, this class of models permits changes in the steady-state real interest rate without corresponding changes in consumption growth, potentially explaining a mismatch in the integration properties of the real interest rate and consumption growth.

### *2.3. Transitional Dynamics*

Section 2.2 discusses factors that can affect the *steady-state* real interest rate. Other shocks can have persistent—but ultimately transitory—effects on the real rate. For example, in the neoclassical growth model, a temporary increase in technology growth or government purchases leads to a persistently (but not permanently) higher real interest rate (Romer, 2006, Chap. 2). In addition, monetary shocks can persistently affect the real interest rate via a variety of frictions, such as “sticky” prices and information, adjustment costs, and learning by agents about policy regimes. Transient technology and fiscal shocks as well as monetary shocks can also explain differences in the persistence of real interest rates and consumption growth. For example, using a calibrated neoclassical equilibrium growth model, Baxter and King (1993) show that a temporary (four-year) increase in government purchases persistently raises the real interest rate, although it eventually returns to its initial level. In contrast, the fiscal shock produces a much less persistent reaction in consumption growth. As we will discuss later, evidence of highly persistent but mean-reverting behavior in real interest rates supports the

empirical relevance of these shocks.

### **3. Testing the Integration Properties of Real Interest Rates**

#### *3.1. Ex Ante versus Ex Post Real Interest Rates*

The *ex ante* real interest rate (EARR) is the nominal interest rate minus the expected inflation rate, while the *ex post* real rate (EPRR) is the nominal rate minus actual inflation. Agents make economic decisions on the basis of their inflation expectations over the decision horizon. For example, the Euler equations (1) and (2) relate the expected marginal utility of consumption to the expected real return. Therefore, the EARR is the relevant measure for evaluating economic decisions, and we really wish to evaluate the EARR's time-series properties, rather than those of the EPRR.

Unfortunately, the EARR is not directly observable because expected inflation is not directly observable. An obvious solution is to use some survey measure of inflation expectations, such as the Livingston Survey of professional forecasters, which has been conducted biannually since the 1940s (Carlson, 1977). Economists are often reluctant, however, to accept survey forecasts as expectations. For example, Mishkin (1981, p. 153) expresses "serious doubts as to the quality of these [survey] data."

There are at least two alternative approaches to the problem of unobserved expectations. The first is to use econometric forecasting methods to construct inflation forecasts; see, for example, Mishkin (1981, 1984) and Huizinga and Mishkin (1986). Unfortunately, econometric forecasting models do not necessarily include all of the relevant information agents use to form expectations of inflation, and such models can fail to change with the structure of the economy.

For example, Stock and Watson (1999, 2003) show that both real activity and asset prices forecast inflation but that the predictive relations change over time.<sup>4</sup>

A second alternative approach is to use the actual inflation rate as a proxy for inflation expectations. By definition, the actual inflation rate at time  $t$  ( $\pi_t$ ) is the sum of the expected inflation rate and a forecast error term ( $\varepsilon_t$ ):

$$\pi_t = E_{t-1}\pi_t + \varepsilon_t. \quad (5)$$

The literature on real interest rates has long argued that, if expectations are formed rationally,  $E_{t-1}\pi_t$  should be an optimal forecast of inflation (Nelson and Schwert, 1977), and  $\varepsilon_t$  should therefore be a white noise process. The EARR can be expressed (approximately) as

$$r_t^{ea} = i_t - E_t\pi_{t+1}, \quad (6)$$

where  $i_t$  is the nominal interest rate. Solving (5) for  $E_t(\pi_{t+1})$  and substituting it into (6), we have

$$r_t^{ea} = i_t - (\pi_{t+1} - \varepsilon_{t+1}) = i_t - \pi_{t+1} + \varepsilon_{t+1} = r_t^{ep} + \varepsilon_{t+1}, \quad (7)$$

where  $r_t^{ep} = i_t - \pi_{t+1}$  is the EPRR. Equation (7) implies that, under rational expectations, the EPRR and EARR differ only by a white noise component, so the EPRR and EARR will share the same long-run (integration) properties. Actually, this latter result does not require expectations to be formed rationally but holds if the expectation errors ( $\varepsilon_{t+1}$ ) are stationary.<sup>5</sup> Beginning with

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<sup>4</sup> Atkeson and Ohanian (2001) and Stock and Watson (2007) discuss the econometric challenges in forecasting inflation. One might also consider using Treasury inflation-protected securities (TIPS) yields—and/or their foreign counterparts—to measure real interest rates. But these series have a relatively short span of available data, in that the U.S. securities were first issued in 1997, are only available at long maturities (5-, 10- and 20-years), and do not correctly measure real rates when there is a significant chance of deflation.

<sup>5</sup> Peláez (1995) provides evidence that inflation-expectation errors are stationary. Also note that Andolfatto et al. (2008) argue that inflation expectations errors can appear serially correlated in finite samples, even when expectations are formed rationally, due to short-run learning dynamics about infrequent changes in the monetary policy regime.

Rose (1988), much of the empirical literature tests the integration properties of the EARR with the EPRR, after assuming that inflation-expectation errors are stationary.

The EPRR's time-series properties can differ from those of the EARR in two ways, however. First, the EPRR's behavior at short horizons might differ from that of the EARR. For example, using survey data and various econometric methods to forecast inflation, Dotsey et al. (2003) study the behavior of the EARR and EPRR at business-cycle frequencies and find that their behavior over the business cycle can differ significantly. Second, some estimation techniques can generate different persistence properties between the EARR and EPRR; see, for example, Evans and Lewis (1995) and Sun and Phillips (2004).

### *3.2. Unit Root and Cointegration Tests: Basic Framework*

This subsection briefly describes the basic framework for unit root and cointegration testing; Hamilton (1994) details the subject. Following Dickey and Fuller (1979) and Said and Dickey (1984), unit root tests are typically based on the autoregressive (AR) representation of a time series, which can be written as follows:

$$y_t - \mu = \rho_1(y_{t-1} - \mu) + \dots + \rho_k(y_{t-k} - \mu) + e_t, \quad (8)$$

where  $e_t$  is a white noise disturbance term. When the sum of the AR coefficients in (8),

$\rho = \sum_{j=1}^k \rho_j$ , equals one, shocks to  $y_t$  persist forever— $y_t$  has a unit root and thus has no

tendency to revert to an unconditional mean. Testing the null hypothesis that  $y_t \sim I(1)$  against

the alternative hypothesis that  $y_t \sim I(0)$  is equivalent to testing  $\sum_{j=1}^k \rho_j = 1$  versus  $\sum_{j=1}^k \rho_j < 1$ .

Researchers usually ignore the possibility that  $\rho > 1$ , since this would imply an explosive

process, which we do not observe in the data. The  $t$ -statistic on  $\gamma$  in the following augmented Dickey-Fuller (ADF) regression provides a convenient test statistic for the unit root null hypothesis:

$$\Delta y_t = \delta + \gamma y_{t-1} + \tilde{\rho}_1 \Delta y_{t-1} + \dots + \tilde{\rho}_{k-1} \Delta y_{t-(k-1)} + e_t, \quad (9)$$

where  $\delta = \mu(1 - \rho)$ ,  $\gamma = -(1 - \rho)$ , and  $\tilde{\rho}_i = -\sum_{j=i+1}^k \rho_j$ .<sup>6</sup> Under the null hypothesis that

$y_t \sim I(1)$ ,  $\gamma = 0$ , while  $\gamma < 0$  under the alternative hypothesis that  $y_t \sim I(0)$ . The  $t$ -statistic on  $\gamma$  in (9) has a non-standard distribution, necessitating simulation methods to obtain critical values.

Cointegration tests are closely related to unit root tests in that they ask whether any linear combination of some set of  $I(1)$  processes (say,  $y_t$  and  $x_t$ ) are stationary or cointegrated. The popular, residual-based augmented Engle and Granger (1987, AEG) procedure uses the following ordinary least squares (OLS) regression as a first step in testing the null hypothesis of no cointegration:

$$y_t = \theta_0 + \theta_1 x_t + u_t. \quad (10)$$

The cointegrating vector, which defines the stable long-run relationship between  $y_t$  and  $x_t$  (if it exists), is given by  $(1, -\theta_1)'$ . One then runs an ADF-type unit root test—with no constant—on the regression residuals,  $\hat{u}_t = y_t - (\hat{\theta}_0 + \hat{\theta}_1 x_t)$ , where  $\hat{\theta}_0$  and  $\hat{\theta}_1$  are the OLS estimates of  $\theta_0$  and  $\theta_1$ .

The AEG test statistic—the ADF test statistic from the residual regression—also has a nonstandard asymptotic distribution, which requires simulated critical values. When  $y_t$  and  $x_t$  are cointegrated,  $\hat{\theta}_0$  and  $\hat{\theta}_1$  are super-consistent, converging to their probability limits faster than the usual rate of  $1/\sqrt{T}$ . Endogeneity bias, however, renders conventional OLS standard errors

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<sup>6</sup> The unit root tests developed by Phillips and Perron (1988) are closely related to ADF tests and are frequently used in the literature. We refer to both ADF and Phillips and Perron (1988) tests simply as ADF tests in our discussion of the empirical literature in Section 3.4 below.

incorrect. When  $y_t$  and  $x_t$  are cointegrated, fully modified OLS (FM-OLS; Phillips and Hansen, 1990) and dynamic OLS (DOLS; Saikkonen, 1991; Stock and Watson, 1993) procedures efficiently estimate  $\theta_0$  and  $\theta_1$  with appropriate standard errors.

Johansen (1991) develops a cointegration test procedure based on the likelihood function of a system of equations that simultaneously tests the null hypothesis of no cointegration and consistently and efficiently estimates the cointegrating vector (if it exists). This system-based approach is also popular in applied research and is potentially more powerful than the single equation-based AEG approach (Pesavento, 2004).

Armed with such econometric procedures, researchers typically evaluate the stationarity of the EPRR with a decision rule. They first analyze the integration properties of the individual components of the EPRR,  $i_t$  and  $\pi_{t+1}$ . If unit root tests indicate that  $i_t$  and  $\pi_{t+1}$  are both  $I(0)$ , then this implies a stationary EPRR, as any linear combination of two  $I(0)$  processes is also an  $I(0)$  process. If  $i_t$  and  $\pi_{t+1}$  have different orders of integration—for example, if  $i_t \sim I(1)$  and  $\pi_{t+1} \sim I(0)$ —then the EPRR must have a unit root, as any linear combination of an  $I(1)$  process and an  $I(0)$  process is an  $I(1)$  process. Finally, if unit root tests show that  $i_t$  and  $\pi_{t+1}$  are both  $I(1)$ , researchers test for a stationary EPRR by testing for cointegration between  $i_t$  and  $\pi_{t+1}$  using one of two approaches. First, many researchers impose a cointegrating vector of  $(1, -1)'$  and apply ADF unit root tests to  $r_t^{ep} = i_t - \pi_{t+1}$ . This approach typically has more power to reject the null of no cointegration when the true cointegrating vector is  $(1, -1)'$ . The second approach is to estimate the cointegrating vector between  $i_t$  and  $\pi_{t+1}$  with the AEG or Johansen (1991) approaches, as this allows for tax effects (Darby, 1975).

If  $i_t, \pi_{t+1} \sim I(1)$ , then a stationary EPRR requires  $i_t$  and  $\pi_{t+1}$  to be cointegrated with cointegrating coefficient  $\theta_1 = 1$  or, allowing for tax effects,  $\theta_1 = 1/(1-\tau)$ , where  $\tau$  is the marginal investor's marginal tax rate on nominal interest income. When allowing for tax effects, researchers view estimates of  $\theta_1$  in the range of 1.3–1.4 as plausible, as they correspond to a marginal tax rate around 0.2–0.3 (Summers, 1983).<sup>7</sup> It is worth emphasizing that cointegration between  $i_t$  and  $\pi_{t+1}$  by itself does not imply a stationary real interest rate:  $\theta_1$  must also equal one [or  $1/(1-\tau)$ ], as other values of  $\theta_1$  imply that the equilibrium real interest rate varies with inflation.

Unit root and cointegration tests have two significant problems. First, they have low power to reject the null if the true model is a highly persistent but stationary process (DeJong et al., 1992). Second, moving-average components in the underlying data-generating process complicate inference from unit root and cointegration tests. Schwert (1987, 1989) shows that ADF unit root tests can have substantial size distortions that lead to spurious rejections of the unit root null hypothesis in the presence of a significant moving-average component.<sup>8</sup> Lütkepohl and Saikkonen (1999) show that such size distortions can also affect cointegration tests. This is potentially relevant when analyzing the EPRR, as Perron and Ng (1996) and others show that inflation rates often have sizable moving-average components.<sup>9</sup>

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<sup>7</sup> Data from tax-free municipal bonds would presumably provide a unitary coefficient. Crowder and Wohar (1999) study the Fisher effect with tax-free municipal bonds.

<sup>8</sup> There are two strategies for dealing with a significant moving-average component in the data-generating process when performing ADF unit root tests: (1) include a large number of lags when estimating (9), as an ARMA process with finite-order lag polynomials can be expressed as an infinite-order AR process; (2) include the moving-average component in the data-generating process when simulating critical values.

<sup>9</sup> Perron (1994) observes that the inflation rate could exhibit a substantial moving-average component if the monetary authority offsets inflationary or disinflationary shocks away from a target price level path.

### 3.3. Early Studies

A collection of early studies on the efficient market hypothesis and the ability of nominal interest rates to forecast the inflation rate foreshadows the studies that employ unit root and cointegration tests. Fama (1975) presents evidence that the monthly U.S. EARR can be viewed as constant over 1953–1971. Nelson and Schwert (1977), however, argue that the statistical tests of Fama (1975) have low power and that his data are actually not very informative about the EARR’s autocorrelation properties. Hess and Bicksler (1975), Fama (1976), Carlson (1977), and Garbade and Wachtel (1978) also challenge Fama’s (1975) finding on statistical grounds. In addition, subsequent studies show that Fama’s (1975) result hinges critically on the particular sample period (Mishkin, 1981, 1984; Huizinga and Mishkin, 1986; Antoncic, 1986).

### 3.4. Unit Root and Cointegration Tests: Empirical Results

The development of unit root and cointegration analysis, beginning with Dickey and Fuller (1979), spurred the studies that formally test the persistence of real interest rates. In his seminal study, Rose (1988) tests for unit roots in short-term nominal interest rates and inflation rates using monthly data for 1947–1986 for 18 OECD countries. Rose (1988) finds that ADF tests fail to reject the null hypothesis of a unit root in short-term nominal interest rates, but can consistently reject a unit root in inflation rates based on various price indices—CPI, GDP deflator, implicit price deflator, WPI. As discussed above, the finding that  $i_t \sim I(1)$  while  $\pi_t \sim I(0)$  indicates that the EPRR,  $i_t - \pi_{t+1}$ , is an  $I(1)$  process. Under the assumption that inflation-expectation errors are stationary, this also implies that the EARR is an  $I(1)$  process.



Rose (1988) easily rejects the unit root null hypothesis for U.S. consumption growth, which leads him to argue that an  $I(1)$  real interest rate and  $I(0)$  consumption growth rate violates the intertemporal Euler equation implied by the consumption-based asset pricing model. Beginning with Rose (1988), Table 1 summarizes the methods and conclusions of papers we survey on the long-run properties of real interest rates.

A number of subsequent papers also test for a unit root in real interest rates. Before estimating structural vector autoregressive (SVAR) models, King et al. (1991) and Galí (1992) apply ADF unit root tests to the U.S. nominal three-month Treasury bill rate, inflation rate, and EPRR. Using quarterly data for 1954–1988 and the GNP deflator inflation rate, King et al. (1991) fail to reject the null hypothesis of a unit root in the nominal interest rate, matching the finding of Rose (1988). Unlike Rose (1988), however, King et al. (1991) cannot reject the unit root null hypothesis for the inflation rate, which creates the possibility that the nominal interest rate and inflation rate are cointegrated. Imposing a cointegrating vector of  $(1, -1)'$ , they fail to reject the unit root null hypothesis for the EPRR. Using quarterly data for 1955–1987, the CPI inflation rate, and simulated critical values that account for potential size distortions due to moving-average components, Galí (1992) obtains unit root test results similar to those of King et al. (1991). Despite the failure to reject the null hypothesis that  $i_t - \pi_{t+1} \sim I(1)$ , Galí (1992) nevertheless assumes that  $i_t - \pi_{t+1} \sim I(0)$  when he estimates his SVAR model, contending that “the assumption of a unit root in the real [interest] rate seems rather implausible on a priori grounds, given its inconsistency with standard equilibrium growth models” (Galí, 1992, p. 717). This is in interesting contrast to King et al. (1991), who maintain the assumption that  $i_t - \pi_{t+1} \sim I(1)$  in their SVAR model. Shapiro and Watson (1988) report similar unit root findings and, like Galí (1992), still assume the EPRR is stationary in an SVAR model.

Analyzing a 1953–1990 full sample as well as a variety of subsamples for the nominal Treasury bill rate and CPI inflation rate, Mishkin (1992) argues that monthly U.S. data are largely consistent with a stationary EPRR. With simulated critical values, as in Galí (1992), Mishkin (1992) finds that the nominal interest rate and inflation rate are both  $I(1)$  over four sample periods: 1953:01–1990:12, 1953:01–1979:10, 1979:11–1982:10, and 1982:11–1990:12. He then tests whether the nominal interest rate and inflation rate are cointegrated using the AEG test and by prespecifying a cointegrating vector and testing for a unit root in  $i_t - \pi_{t+1}$ . Mishkin (1992) rejects the null hypothesis of no cointegration for the 1953:01–1990:12 and 1953:01–1979:10 periods, but finds less frequent and weaker rejections for the 1979:11–1982:10 and 1982:11–1990:12 periods.<sup>10</sup> Mishkin and Simon (1995) apply similar tests to quarterly short-term nominal interest rate and inflation rate data for Australia. Using a 1962:3–1993:4 full sample, as well as 1962:3–1979:3 and 1979:4–1993:4 subsamples, they find evidence that both the nominal interest rate and the inflation rate are  $I(1)$ , agreeing with the results for U.S. data in Mishkin (1992). There is weaker evidence that the Australian nominal interest rate and inflation rate are cointegrated than there is for U.S. data. Nevertheless, Mishkin and Simon (1995) argue that theoretical considerations warrant viewing the long-run real interest rate as stationary in Australia, as “any reasonable model of the macro economy would surely suggest that real interest rates have mean-reverting tendencies which make them stationary” (Mishkin and Simon, 1995, p. 223).

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<sup>10</sup> Despite the fact that they use essentially the same econometric procedures and similar samples, Galí (1992) is unable to reject the unit root null hypothesis for the EPRR, while Mishkin (1992) does reject this null hypothesis. This illustrates the sensitivity of EPRR unit root and cointegration tests to the specific sample. In addition, the use of short samples, such as the 1979:11–1982:10 sample period considered by Mishkin (1992), is unlikely to be informative about the integration properties of the EPRR. To infer long-run behavior, one needs reasonably long samples.

Kousta and Serletis (1999) test for unit roots and cointegration in short-term nominal interest rates and CPI inflation rates using quarterly data for 1957–1995 for eleven industrialized countries. They employ ADF unit root tests as well as the KPSS unit root test of Kwiatkowski et al. (1992), which takes stationarity as the null hypothesis and nonstationarity as the alternative. ADF and KPSS unit root tests indicate that  $i_t \sim I(1)$  and  $\pi_t \sim I(1)$  in most countries, so a stationary EPRR requires cointegration between the nominal interest rate and inflation rate. Kousta and Serletis (1999), however, usually fail to find strong evidence of cointegration using the AEG test. Overall, their study finds that the EPRR is nonstationary in many industrialized countries. Rapach (2003) obtains similar results using postwar data for an even larger number of OECD countries.

In a subtle variation on conventional cointegration analysis, Bierens (2000) allows an individual time series to have a deterministic component that is a highly complex function of time—essentially a smooth spline—and a stationary stochastic component, and he develops nonparametric procedures to test whether two series share a common deterministic component (“nonlinear cotrending”). Using monthly U.S. data for 1954–1994, Bierens (2000) presents evidence that the federal funds rate and CPI inflation rate cotrend with a vector of  $(1, -1)'$ , which can be interpreted as evidence for a stationary real interest rate. Bierens (2000) shows, however, that his tests cannot differentiate between nonlinear cotrending and linear cointegration in the presence of stochastic trends in the nominal interest rate and inflation rate. In essence, the highly complex deterministic components for the individual series closely mimic unit root behavior.

A number of studies use the Johansen (1991) system-based cointegration procedure to test for a stationary EPRR. Wallace and Warner (1993) apply the Johansen (1991) procedure to quarterly U.S. nominal three-month Treasury bill rate and CPI inflation data for a 1948–1990 full

sample and a number of subsamples. Their results generally support the existence of a cointegrating relationship, and their estimates of  $\theta_1$  are typically not significantly different from unity, in line with a stationary EPRR. Wallace and Warner (1993) also argue that the expectations hypothesis implies that short-term and long-term nominal interest rates should be cointegrated, and they find evidence that U.S. short and long rates are cointegrated with a cointegrating vector of  $(1, -1)'$ . In line with the results for the nominal three-month Treasury bill rate, Wallace and Warner (1993) find that the nominal ten-year Treasury bond rate and inflation rate are cointegrated.

Using quarterly U.S. data for 1951–1991, Crowder and Hoffman (1996) also employ the Johansen (1991) procedure to test for cointegration between the three-month Treasury bill rate and implicit consumption deflator inflation rate. As in Wallace and Warner (1993), they reject the null of no cointegration between the nominal interest rate and inflation rate. Their estimates of  $\theta_1$  range from 1.22–1.34, which are consistent with a stationary tax-adjusted EPRR. Crowder and Hoffman (1996) also use estimates of average marginal tax rates to directly test for cointegration between  $i_t(1 - \tau_t)$  and  $\pi_{t+1}$ . The Johansen (1991) procedure supports cointegration and estimates a cointegrating vector not significantly different from  $(1, -1)'$ , in line with a stationary tax-adjusted EPRR.

Engsted (1995) uses the Johansen (1991) procedure to test for cointegration between the nominal long-term government bond yield and CPI inflation rate in 13 OECD countries using quarterly data for 1962–1993. In broad agreement with the results of Wallace and Warner (1993) and Crowder and Hoffman (1996), Engsted (1995) rejects the null hypothesis of no cointegration for almost all countries. The estimates of  $\theta_1$  vary quite markedly across countries, however, and the values are often inconsistent with a stationary EPRR.

Overall, unit root and cointegration tests present mixed results with respect to the integration properties of the EPRR. Generally speaking, single-equation methods provide weaker evidence of a stationary EPRR, while the Johansen (1991) system-based approach supports a stationary EPRR, at least for the U.S. Unfortunately, econometric issues, such as the low power of unit root tests and size distortions in the presence of moving-average components, complicate inference about persistence.

To address these econometric issues, Rapach and Weber (2004) employ unit root and cointegration tests with improved size and power. Specifically, they use the Ng and Perron (2001) unit root and Perron and Rodriguez (2001) cointegration tests. These tests incorporate aspects of the modified ADF tests in Elliott et al. (1996) and Perron and Ng (1996), as well as an adjusted modified information criterion to select the AR lag order, to develop tests that avoid size distortions while retaining power. Rapach and Weber (2004) use quarterly nominal long-term government bond yield and CPI inflation rate data for 1957–2000 for 16 industrialized countries. The Ng and Perron (2001) unit root and Perron and Rodriguez (2001) cointegration tests provide mixed results, but Rapach and Weber (2004) interpret their results as indicating that the EPRR is nonstationary in most industrialized countries over the postwar era.

### *3.5. Updated Unit Root and Cointegration Test Results for U.S. Data*

Tables 2 and 3 illustrate the type of evidence provided by unit root and cointegration tests for the U.S. three-month Treasury bill rate, CPI inflation rate, and per capita consumption growth rate for 1953:1–2007:2 (the same data as in Figure 1).

Table 2 reports the ADF statistic as well as the  $MZ_\alpha$  statistic from Ng and Perron (2001), which is designed to have better size and power properties than the former. Consistent with the literature, neither test rejects the unit root null hypothesis for the nominal interest rate. The results are mixed for the inflation rate: the ADF statistic rejects the unit root null at the 10% level, but the  $MZ_\alpha$  statistic does not reject at conventional significance levels. The ADF test result that  $i_t \sim I(1)$  while  $\pi_t \sim I(0)$  means that the EPRR is nonstationary, as in Rose (1988).<sup>11</sup> The  $MZ_\alpha$  statistic's failure to reject the unit root null for either inflation or nominal interest rates argues for cointegration analysis of those variables to ascertain the EPRR's integration properties. When we prespecify a  $(1, -1)'$  cointegrating vector and apply unit root tests to the EPRR, we reject the unit root null at the 5% level using the ADF statistic and 1% level using the  $MZ_\alpha$  statistic. The U.S. EPRR appears to be stationary.

To test the null hypothesis of no cointegration without prespecifying a cointegrating vector, Table 3 reports the AEG statistic,  $MZ_\alpha$  statistic from Perron and Rodriguez (2001), and trace statistic from Johansen (1991). The AEG and trace statistics reject the null hypothesis of no cointegration at the 10% level, and the  $MZ_\alpha$  statistic rejects the null at the 5% level. Table 3 also reports estimates of the cointegrating coefficients,  $\theta_0$  and  $\theta_1$ , in (10). Neither the DOLS nor Johansen (1991) estimates of  $\theta_1$  are significantly different from unity, indicating a stationary U.S. EPRR. The cointegrating vector is not estimated precisely enough to determine whether there is a tax effect.

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<sup>11</sup> A significant moving-average component in the inflation rate could create size distortions in the ADF statistic that lead us to falsely reject the unit root null hypothesis for that series. The fact that we do not reject the unit root null using the  $MZ_\alpha$  statistic—which is designed to avoid this size distortion—supports this interpretation. Rapach and Weber (2004), however, do reject the unit root null for the U.S. inflation rate using the  $MZ_\alpha$  statistic and data through 2000. Inflation rate unit root tests are thus particularly sensitive to the sample period.

Tables 2 and 3 provide evidence that the U.S. EPRR is stationary, although some of the rejections are marginal. Unit root and cointegration test results, however, are sensitive to the test procedure and sample period. Studies such as Mishkin (1992), Wallace and Warner (1993), and Crowder and Hoffman (1996) find evidence of a stationary U.S. EPRR, but Koustas and Serletis (1999) and Rapach and Weber (2004) generally do not. In contrast, per capita consumption growth is clearly stationary, as the ADF and  $MZ_\alpha$  statistics in Table 2 both strongly reject the unit root null hypothesis for this variable. The fact that integration tests give mixed results for the EPRR's stationarity and clear-cut results for consumption growth highlights differences in the persistence properties of the two variables.

### *3.6. Confidence Intervals for the Sum of the AR Coefficients*

It is inherently difficult to distinguish an  $I(1)$  process from a highly persistent  $I(0)$  process, as the two types of processes can be observationally equivalent (Blough, 1992; Faust, 1996).<sup>12</sup> To analyze the theoretical implications of the time-series properties of the real interest rate, however, we want to determine a range of values for  $\rho$  in (8) that are consistent with the data, not only whether  $\rho$  is less than or equal to one. That is, a series with a  $\rho$  value of 0.95 is highly persistent, even if it does not contain a unit root *per se*, and it is much more persistent than a series with a  $\rho$  value of, say, 0.4.

To calculate the degree of persistence in the data—rather than simply trying to determine if the series is  $I(0)$  or  $I(1)$ —Rapach and Wohar (2004) compute 95% confidence intervals for

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<sup>12</sup> In line with this, Crowder and Hoffman (1996) emphasize that impulse response analysis indicates that shocks have very persistent effects on the EPRR, although the U.S. EPRR appears to be  $I(0)$ .

$\rho$  using the Hansen (1999) grid bootstrap and Romano and Wolf (2001) subsampling procedures.<sup>13</sup> Using quarterly nominal long-term government bond yield and CPI inflation rate data for 13 industrialized countries for 1960–1998, Rapach and Wohar (2004) report that the lower bounds of the 95% confidence interval for  $\rho$  for the tax-adjusted EPRR are often greater than 0.90, while the upper bounds are almost all greater than unity. Similarly, Karanasos et al. (2006) use a very long span of monthly U.S. long-term government bond yield and CPI inflation data for 1876–2000 to compute a 95% confidence interval for the EPRR's  $\rho$ . Their computed interval, [0.97, 0.99], indicates that the U.S. EPRR is a highly persistent or near-unit-root process, even if it does not actually contain a unit root.

Using the same U.S. data underlying the results in Tables 2 and 3, we employ the Hansen (1999) grid bootstrap and Romano and Wolf (2001) subsampling procedures to compute a 95% confidence interval for  $\rho$  in the  $i_t - \pi_{t+1}$  process. The grid-bootstrap and subsampling confidence intervals are [0.77, 0.97] and [0.71, 0.97], and the upper bounds are consistent with a highly persistent process. In contrast, the grid bootstrap and subsampling 95% confidence intervals for  $\rho$  for per capita consumption growth are [0.34, 0.70] and [0.37, 0.64]. The upper bounds of the confidence intervals for  $\rho$  for consumption growth are less than the lower bounds of the confidence intervals for  $\rho$  for the EPRR. This is another way to characterize the mismatch in the persistence properties of the EPRR and consumption growth.

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<sup>13</sup> Andrews and Chen (1994) argue that the sum of the AR coefficients,  $\rho$ , characterizes the persistence in a series, as it is related to the cumulative impulse response function and the spectrum at zero frequency. While conventional asymptotic or bootstrap confidence intervals do not generate valid confidence intervals for nearly integrated processes (Basawa et al., 1991), Hansen (1999) and Romano and Wolf (2001) show that their procedures do generate confidence intervals for  $\rho$  with correct first-order asymptotic coverage. Mikusheva (2007) shows, however, that while the Hansen (1999) grid-bootstrap procedure has correct asymptotical coverage, the Romano and Wolf (2001) subsampling procedure does not.



### 3.7. Testing for Fractional Integration

Unit root and cointegration tests are designed to ascertain whether a series is  $I(0)$  or  $I(1)$ , and the  $I(0)/I(1)$  distinction implicitly restricts—perhaps inappropriately—the types of dynamic processes allowed. In response, some researchers test for fractional integration (Granger, 1980; Granger and Joyeux, 1980; Hosking, 1981) in the EARR and EPRR. A fractionally integrated series is denoted by  $I(d)$ ,  $0 \leq d \leq 1$ . When  $d = 0$ , the series is  $I(0)$ , and shocks die out at a geometric rate; when  $d = 1$ , the series is  $I(1)$ , and shocks have permanent effects or “infinite memory.” An intermediate case occurs when  $0 < d < 1$ : the series is mean-reverting, as in the  $I(0)$  case, but shocks now die out at a much slower hyperbolic (rather than geometric) rate. Series where  $0 < d < 1$  exhibit “long memory,” mean-reverting behavior and can be substantially more persistent than even a highly persistent  $I(0)$  series.

A number of studies, including Lai (1997), Tsay (2000), Karanasos et al. (2006), Sun and Phillips (2004), and Pipatchaipoom et al. (2005), test for fractional integration in the U.S. EPRR or EARR. Using U.S. postwar monthly or quarterly U.S. data, Lai (1997), Tsay (2000), and Pipatchaipoom et al. (2005) all present evidence of long-memory, mean-reverting behavior, as estimates of  $d$  for the U.S. EPRR or EARR typically range from 0.7–0.8 and are significantly above zero and below one. Using a long span of annual U.S. data (1876–2000), Karanasos et al. (2006) similarly find evidence of long-memory, mean-reverting behavior in the EPRR. Sun and Phillips (2004) develop a new bivariate econometric procedure that estimates the EARR’s  $d$  parameter in the 0.75–1.0 range for quarterly postwar U.S. data.

Overall, fractional integration tests indicate that the U.S. EPRR and EARR do not contain a unit root *per se*, but that these variables are mean-reverting and very persistent. We confirm

this by estimating  $d$  for the EPRR using our sample of U.S. data for 1953:1–2007:2 with the Shimotsu (2008) semiparametric two-step feasible exact local Whittle estimator that allows for an unknown mean in the series. This estimator refines the Shimotsu and Phillips (2005) exact local Whittle estimator, and these authors show that such local Whittle estimators of  $d$  have good properties in Monte Carlo experiments. The estimate of  $d$  for the EPRR is 0.71, with a 95% confidence interval of [0.51, 0.90], so we can reject the hypothesis that  $d = 0$  or  $d = 1$ . This evidence of long-memory, mean-reverting behavior is consistent with the results from the literature discussed above. The estimate of  $d$  for per capita consumption growth is 0.15 with a standard error of 0.10, so we cannot reject the hypothesis that  $d = 0$  at conventional significance levels. This is another manifestation of the discrepancy in persistence between the real interest rate and consumption growth.

### *3.8. Testing for Threshold Dynamics and Nonlinear Cointegration*

The empirical literature on the real interest rate typically employs models that assume both the cointegrating relationship and short-run dynamics to be linear.<sup>14</sup> Recently, researchers have begun to relax these linearity assumptions in favor of nonlinear cointegration or threshold dynamics, which allow for the cointegrating relationship or mean reversion to depend on the current values of the variables. For example, a threshold model might permit the EPRR to be approximately a random walk within  $\pm 2\%$  of some long-run equilibrium value but to revert strongly to the  $\pm 2\%$  bands when it wanders outside of the bands.<sup>15</sup>

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<sup>14</sup> Studies that allow for fractional integration or structural breaks also relax some linearity assumptions, but in a different way than those reviewed in this subsection.

<sup>15</sup> The purchasing power parity literature often uses these threshold models (Sarno and Taylor, 2002).

Million (2004) presents evidence that the U.S. EPRR adjusts in a nonlinear fashion to a long-run equilibrium level using a logistic smooth transition autoregressive (LSTAR) model and monthly U.S. three-month Treasury bill rate and CPI inflation rate data for 1951–1999. The Lagrange multiplier test of Luukkonen et al. (1988) rejects the null hypothesis of a linear dynamic adjustment process, and there is evidence of stronger (weaker) mean reversion in the EPRR for values of the EPRR below (above) a threshold level of 2.2%. Million (2004) notes that the weak mean reversion in the upper regime is consistent with the fact that the U.S. real interest rate was persistently high during much of the 1980s, and he observes that the Federal Reserve’s priority on fighting inflation, following the stagflation of the 1970s, could explain this period of high real rates. In a vein similar to that of Million (2004), Koustas and Lamarche (2008) estimate three-regime self-exciting threshold autoregressive (SETAR) models to characterize the monetary policy strategy of “opportunistic disinflation” (Blinder, 1994; Orphanides and Wilcox, 2002). Based on the Bec et al. (2004) nonlinear unit root and Hansen (1996, 1997) linearity tests, Koustas and Lamarche (2008) conclude that the EPRR can be suitably modeled as a three-regime SETAR process in Canada, France, and Italy over the postwar period.<sup>16</sup>

Christopoulos and León-Ledesma (2007) examine quarterly U.S. three-month Treasury bill rate and CPI inflation rate data for 1960–2004, permitting the cointegrating relationship itself to be nonlinear. More precisely, they allow the cointegrating coefficient ( $\theta_1$ ) to vary with the inflation rate by estimating logistic and smooth exponential transition regression (LSTR and ESTR) models. Christopoulos and León-Ledesma (2007) find significant evidence of nonlinear cointegration between the nominal interest rate and inflation rate using the Choi and Saikkonen

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<sup>16</sup> Maki (2003) uses the Breitung (2002) nonparametric procedure that allows for nonlinear adjustment dynamics to test for cointegration between the Japanese nominal interest rate and CPI inflation rate for 1972:1–2000:12. While Maki (2003) finds significant evidence of cointegration between the nominal interest rate and inflation rate using the Breitung (2002) test, he does not estimate the cointegrating vector, so it is not clear that the long-run equilibrium relationship is consistent with a stationary EPRR.

(2005) test. Employing estimation techniques from Saikkonen and Choi (2004), the authors conclude that the ESTR model fits best over the full sample (1960:1–2004:4) and the first subsample (1960:1–1978:1), while the LSTR model fits best over the second subsample (1979:1–2004:4). The estimated ESTR model for 1960:1–1978:1 is not consistent with a stationary real EPRR for any inflation rate, while the estimated LSTR model for 1979:1–2004:4 is consistent with a stationary EPRR only when the inflation rate moves above approximately 3%.

In summary, recently developed econometric procedures provide some evidence of threshold behavior or nonlinear cointegration in the EPRR in certain industrialized countries. In some cases, the threshold models accord well with our intuition about changes in central bank policies. While evidence of threshold behavior in real interest rates is potentially interesting, the models do not obviate the persistence in real interest rates, as there are still regimes where the real interest rate behaves very much like a unit root process.

#### **4. Testing for Regime Switching and Structural Breaks in Real Interest Rates**

Building on Huizinga and Mishkin (1986), another strand of the empirical literature tests for structural breaks in real interest rates. Accounting for such breaks can substantially reduce the persistence within the regimes defined by those breaks (Perron, 1989). Similarly, failing to account for structural breaks can produce spurious evidence of fractional integration (Jouini and Nouira, 2006).

Using quarterly U.S. three-month Treasury bill rate and CPI inflation rate data for 1961–1986, Garcia and Perron (1996) employ Hamilton’s (1989) Markov-switching approach to test for regime shifts in the U.S. EPRR. Specifically, they allow the unconditional mean of an AR(2) process— $\mu$  in (8) with  $k = 2$ —to follow a three-state Markov process. The three estimated

states correspond to high, middle, and low regimes with means of approximately 5.5%, 1.4%, and -1.8%. The filtered probability estimates show that the EPRR was likely in the middle regime from 1961–1973, the low regime from 1973–1981, and the high regime from 1981–1986. There is very little persistence within each regime, as the estimated AR coefficients [ $\rho_1$  and  $\rho_2$  in (8)] are near zero within regimes. Overall, Garcia and Perron (1996) argue that the U.S. real interest rate occasionally experiences sizable shifts in its mean value, while the real interest rate is close to constant within the regimes.

Applications of Markov-switching models typically assume that the model is ergodic, so the current state will eventually cycle back to any possible state. Structural breaks have some similar properties to Markov-switching regimes, but they are not ergodic—they do not necessarily tend to revert to previous conditions. Because real interest rates in Garcia and Perron (1996) exhibit no obvious tendency to return to previous states, one might think structural breaks would be more appropriate for modeling real interest rate changes than Markov switching. Bai and Perron (1998) develop a powerful methodology for testing for multiple structural breaks in a regression model, and Caporale and Grier (2000) and Bai and Perron (2003) apply this methodology to the mean of the U.S. EPRR. Both studies use quarterly U.S. short-term nominal interest rate and CPI inflation rate data for 1961–1986, and the estimated break dates are very similar: 1967:1, 1972:4, and 1980:2 in Caporale and Grier (2000) and 1966:4, 1972:3, and 1980:3 in Bai and Perron (2003). The breaks correspond to a decrease in the mean EPRR in 1966/1967, a further decrease in 1972, and a sharp increase in 1980. Caporale and Grier (2000) argue that changes in political regimes—party control of the presidency and Senate—produce these regime changes.

Rapach and Wohar (2005) extend Caporale and Grier (2000) and Bai and Perron (2003) by applying the Bai and Perron (1998) methodology to the EPRR in 13 industrialized countries using tax-adjusted nominal long-term government bond yield and CPI inflation rate data for 1960–1998. They find significant evidence of structural breaks in the mean of the EPRR in each of the 13 countries. Rapach and Wohar (2005) also find that breaks in the mean inflation rate often coincide with breaks in the mean EPRR for each country's data. Furthermore, increases (decreases) in the mean inflation rate are almost always associated with decreases (increases) in the mean EPRR. This finding is consistent with the hypothesis that monetary easing increases inflation and generates a persistent decline in the real interest rate.

In a comment on Rapach and Wohar (2005), Caporale and Grier (2005) examine whether political regime changes affect the mean U.S. EPRR, after controlling for the effects of regime changes in the inflation rate. Caporale and Grier (2005) find that political regime changes associated with changes in the party of the president or control of Congress do not affect the mean EPRR after controlling for inflation. However, the appointments of Federal Reserve Chairmen Paul Volcker in 1979 and Alan Greenspan in 1987 are associated with shifts in the mean EPRR even after controlling for changes in the mean inflation rate.

The previous papers test for structural breaks under the assumption of stationary within-regime behavior. In the spirit of Perron (1989), a number of studies test whether the real interest rate is  $I(0)$  after allowing for deterministic shifts in the mean real rate. Extending the methodology of Perron and Vogelsang (1992), Clemente et al. (1998) test the unit root null hypothesis for the U.K. and U.S. EPRR using quarterly long-term government bond yield and CPI inflation rate data for 1980–1995, allowing for two breaks in the mean of the EPRR. They find that the EPRR in the U.K. and U.S. is an  $I(0)$  process around an unconditional mean with

two breaks. Using monthly U.S. one-year Treasury bill rate data for 1978–2002 as well as expected inflation data from the University of Michigan’s *Survey of Consumers*, Lai (2004) finds that the EARR is an  $I(0)$  process with a shift in its unconditional mean in the early 1980s. Lai (2007) extends Lai (2004) by allowing for a mean shift in quarterly real interest rates for eight industrialized countries and eight developing countries and finds widespread support for a stationary EPRR after allowing for a break in the unconditional mean.

To further illustrate the prevalence of structural breaks, we use the Bai and Perron (1998) methodology to test for such instability in the unconditional mean of the U.S. EPRR for 1953:1–2007:2. Table 4 reports the results. The procedure finds three changes in the mean, which occur at 1972:3, 1980:3, and 1989:3 and are similar to those previously identified for the U.S.<sup>17</sup> The breaks are associated with substantial changes in the average annualized real interest rate in the different regimes: the average real rate is 1.22% for 1953:1–1972:3, is not significantly different from zero for 1972:4–1980:3, increases to 4.58% for 1980:4–1989:3, and falls to 1.82% for 1989:4–2007:2. Figure 2 depicts the EPRR along with the mean for each of the four regimes defined by the three breaks.<sup>18</sup> In contrast to this evidence for breaks in the real rate, the Bai and Perron (1998) methodology fails to discover significant evidence of structural breaks in the mean of per capita consumption growth. (We omit complete results for brevity.)

In interpreting structural break results, we emphasize that such breaks only reduce within-regime or local persistence in real interest rates. The existence of breaks still implies a high degree of global persistence, and the breaks themselves require an economic explanation.

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<sup>17</sup> Rapach and Wohar (2005) discuss how the statistics reported in Table 4 imply that there are three significant breaks in the unconditional mean.

<sup>18</sup> The test results of Bai and Perron (1998) for structural breaks in the mean EPRR do not appear sensitive to whether the tax-adjusted or tax-unadjusted EPRR is used (Rapach and Wohar, 2005). Neither do estimates of the sum of the AR coefficients nor tests for fractional integration hinge critically on whether the EPRR is tax-adjusted.

## 5. Theoretical Implications and a Monetary Explanation of Persistence

This section considers what types of shocks are most likely to produce the persistence in the U.S. real interest rate. The empirical literature devotes relatively little attention to this important issue. We argue that monetary shocks likely drive the persistence in the U.S. real interest rate.

Before discussing potential sources of real interest rate persistence, we briefly make the case that the U.S. real interest rate is ultimately mean-reverting. As we emphasize, unit root and cointegration tests have difficulty distinguishing unit root processes from persistent but stationary alternatives. Nevertheless, unit root and cointegration tests with good size and power, applied to updated data, provide evidence that the U.S. real interest rate is an  $I(0)$ —and thus mean-reverting—process (Table 2).<sup>19</sup> Tests for fractional integration nest the  $I(0)/I(1)$  alternatives, and they concur that the U.S. real interest rate is a mean-reverting process. Using an updated sample, we confirm the findings of Lai (1997), Tsay (2000), Pipatchaipoom et al. (2005) and Karanasos et al. (2006) that demonstrate long memory, mean-reverting behavior in the U.S. real interest rate. Our updated sample also provides evidence of structural breaks in the U.S. real interest rate. Curiously, the regime-specific mean breaks for the EPRR largely cancel each other out in the long run (Table 4): the estimated mean real rate in 2007 is close to that estimated for 1953.<sup>20</sup> Structural breaks thus appear to exhibit a certain type of mean-reverting behavior.

These facts lead us to tentatively claim that the U.S. real interest rate is best viewed as a very persistent but ultimately mean-reverting process. We emphasize the tentative nature of this

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<sup>19</sup> Recall, however, that unit root and cointegration tests are sensitive to the particular sample employed.

<sup>20</sup> One might wonder if the observed mean reversion in structural breaks contradicts our contention that the breaks should not be modeled as a Markov process because they are not ergodic. We do not think, however, that observing one state twice and two states once provides sufficient information for a Markov process.



claim, and we consider careful econometric testing of this proposition to be an important area for future research. Even if real interest rates ultimately mean-revert, they are clearly very persistent.

Recall the underlying motivation for learning about real interest rate persistence: In a simple endowment economy, the real interest rate should have the same persistence properties as consumption growth. In fact, however, real rates are much more persistent than consumption growth. Permanent technology growth shocks can create a nonstationary real rate but affect consumption growth in the same way, so they cannot account for the mismatch in persistence. More complex equilibrium growth models potentially explain this persistence mismatch through changing fiscal and monetary policy as well as transient technology growth shocks. We consider fiscal, monetary, and transient technology shocks as potential causes of persistent fluctuations in the U.S. real interest rate.

Figures 1 and 2 reveal two episodes of pronounced and prolonged changes in the U.S. EPRR: the protracted decrease in the EPRR in the 1970s and subsequent sharp increase in the 1980s. Fiscal shocks appear to be an unlikely explanation for the large decline in real rates from 1972 to 1979. The U.S. did not undertake the sort of contractionary fiscal policy that would be necessary for such a fall in real rates. In fact, fiscal policy in the 1970s largely tended toward modest deficits. Given the substantial budget deficits beginning in 1981, expansionary fiscal shocks are a more plausible candidate for the increase in real rates at this time.

Monetary shocks appear to fit well with the overall pattern in the real interest rate, including the multi-year decline in the real rate during the 1970s, the very sharp 1980 increase, and subsequent gradual decline during the “Great Disinflation.” One interpretation of the “Great Inflation” that began in the late 1960s and lasted throughout the 1970s is that the Federal Reserve pursued an expansionary monetary policy—either inadvertently or to reduce the unemployment

rate to unsustainable levels—and this persistently reduced the real interest rate (Delong, 1997; Barsky and Kilian, 2002; Meltzer, 2005; Romer, 2005). After Paul Volcker’s appointment as Chairman, the Federal Reserve sharply raised short-term nominal interest rates to reduce inflation from its early 1980 peak of nearly 12%, and this produced a sharp and prolonged increase in the real interest rate. The structural breaks manifest these pronounced swings: the mean EPRR falls from 1.22% in 1972:3 to essentially zero and then rises to 4.58% beginning in 1980:4 (Table 4). Furthermore, Rapach and Wohar (2005) report evidence of breaks in the mean U.S. inflation rate in 1973:1 and 1982:1 that increase and decrease the average inflation rate. The timing and direction of the breaks are consistent with a monetary explanation that also accounts for the mismatch in persistence between the real interest rate and consumption growth. In each case, negative (positive) breaks to the real rate of interest coincide with positive (negative) breaks in the mean rate of inflation. The data are in line with the hypothesis that central banks change monetary policy and inflation through persistent effects on the real rate of interest.

Turning to technology shocks, the paucity of independent data on technology shocks makes it difficult to correlate such changes with real interest rates. In addition, researchers have traditionally viewed technology growth as reasonably stable. One might think that other sorts of supply shocks, such as oil price increases, might influence the real rate, and they surely do to some degree; Barro and Sala-i-Martin (1990) and Caporale and Grier (2000), for example, consider this possibility. It is unlikely, however, that oil price shocks alone can account for the pronounced swings in the U.S. real interest rate: Why would rising oil prices in 1973 reduce the real interest rates but rising oil prices in 1979 dramatically raise the real rate?<sup>21</sup>

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<sup>21</sup> Furthermore, Barsky and Kilian (2002) argue that the timing of increases in U.S. inflation in the early 1970s is more consistent with a monetary rather than an oil price shock explanation.

While we interpret the timing of major swings in the U.S. real rate to strongly suggest a monetary explanation, we ultimately need to estimate structural models to analyze the relative importance of various shocks. Galí (1992) is one of the few studies providing evidence on the economic sources of real interest rate persistence. His SVAR model finds that an expansionary money supply shock leads to a very persistent decline in the real interest rate, and money supply shocks account for nearly 90% of the variance in the real rate at the one-quarter horizon and still account for around 60% of the variance at the 20-quarter horizon. Galí's (1992) evidence is consistent with our monetary explanation of real interest rate persistence.<sup>22</sup>

We present additional evidence in support of a monetary explanation of real interest rate persistence based on the new measure of monetary shocks developed by Romer and Romer (2004). They cull through quantitative and narrative Federal Reserve records to compute a monetary policy shock series for 1969–1996 that is independent of systematic responses to anticipated economic conditions. Figure 3 plots the Romer and Romer (2004) monetary policy shocks series, where expansionary (i.e., negative) shocks in the late 1960s and early 1970s and large contractionary (i.e., positive) shocks in the late 1970s and early 1980s appear to match well with the decline in the U.S. real interest rate in the 1970s and subsequent sharp increase around 1980.

Romer and Romer (2004) estimate autoregressive distributed lag (ARDL) models to examine the effects of a monetary policy shock on real output and the price level. They find that a contractionary shock creates persistent and sizable declines in both real output and the price level. In similar fashion, we estimate an ARDL model via OLS to measure the effects of a monetary policy shock on the real interest rate. The ARDL model takes the form,

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<sup>22</sup> King and Watson (1997) and Rapach (2003) use SVAR frameworks to estimate the long-run effects of exogenous changes in inflation on the real interest rate. Both studies find evidence that an exogenous increase in the steady-state inflation rate decreases the steady-state real interest rate.

$$r_t^{ep} = a_0 + \sum_{j=1}^8 a_j r_{t-j}^{ep} + \sum_{j=0}^8 b_j S_{t-j} + u_t, \quad (11)$$

where  $r_t^{ep}$  is the EPRR and  $S_t$  is the Romer and Romer measure of monetary policy shocks.

Figure 4 illustrates the response of the EPRR to a monetary policy shock of size 0.5, which is comparable to some of the contractionary shocks experienced in the late 1970s and early 1980s (Figure 3). Romer and Romer's (2004) Monte Carlo methods provide the two-standard-error bands. A contractionary monetary policy shock produces a statistically and economically significant increase in the U.S. EPRR, which remains statistically significant after approximately two years. Note that the response in Figure 4 is nearly identical to the response of  $r_t^{ep}$  to a shock to  $S_t$  obtained from a bivariate VAR(8) model that orders  $S_t$  first in a Cholesky decomposition. Together, Figures 3 and 4 show that expansionary (contractionary) monetary policy shocks can account for the pronounced and prolonged decrease (increase) in the U.S. real interest rate in the 1970s (early 1980s).

Of course, structural identification is a thorny issue, and more research is needed to determine the veracity of the monetary explanation for U.S. real interest rate persistence.

## 6. Conclusion

Rose's (1988) seminal study spurred a sizable empirical literature that examines the time-series properties of real interest rates. Our survey details the evidence that real interest rates are highly persistent. This persistence manifests itself in the following ways:

- Many studies indicate that real interest rates contain a unit root. While econometric problems prevent a dispositive resolution of this question, real interest rates display

behavior that is very persistent, close to a unit root.

- Estimated 95% confidence intervals for the sum of the AR coefficients from the literature have upper bounds that are greater than or very near unity.
- Real interest rates appear to display long memory behavior; shocks are very long-lived, but the real interest rate is estimated to be ultimately mean-reverting.
- Studies allowing for nonlinear dynamics in real interest rates identify regimes where the real interest behaves like a unit root process.
- Structural breaks in unconditional means characterize real interest rates. While the breaks reduce within-regime persistence, the real interest rate remains highly persistent overall.

Although researchers have used a variety of econometric models to analyze the time-series properties of real interest rates, relatively little work has been done to discriminate among these sundry models. Model selection could tell us, for example, whether we should think of persistent changes in real interest rates in terms of changes in the steady-state real rate—which are consistent with unit root behavior—or long-lived shocks that eventually decay to a stable steady-state real rate—which are consistent with mean-reverting behavior. While model selection raises challenging econometric (and philosophical) issues, out-of-sample forecasting exercises and analysis of posterior model probabilities in a Bayesian context might identify the best way to model real interest rate persistence.

Finally, structural analysis is necessary to identify the sources of the persistence in real interest rates. Theoretical models suggest that a variety of shocks can induce real rate persistence, including preference, technology growth, fiscal, and monetary shocks. We suggest a monetary explanation of U.S. real interest rate persistence based on timing, lack of persistence in consumption growth, and large and persistent real interest rate responses to a Romer and Romer

(2004) monetary policy shock. The literature would greatly benefit from further analysis of the relative importance of different types of shocks in explaining real interest rate persistence.

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Table 1: Selective summary of the empirical literature on the long-run properties of real interest rates

Study	Sample	Countries	Nominal interest rate and price data	Results on the long-run properties of nominal interest rates, inflation rates, and real interest rates
Rose (1988)	A: 1892–1970, 1901–1950 Q: 1947–1986 M: 1948–1986	18 OECD countries	Long-term corporate bond yields, short-term commercial paper rates, GNP deflator, CPI, implicit price deflator, WPI	ADF tests fail to reject a unit root for nominal interest rates but do reject for inflation rates, indicating a unit root in EPRRs. ADF tests do reject a unit root for consumption growth.
King et al. (1991)	Q: 1949–1988	U.S.	3-month U.S. Treasury bill rate, implicit GNP deflator	ADF tests fail to reject a unit root for the nominal interest rate, inflation rate, and EPRR.
Galí (1992)	Q: 1955–1987	U.S.	3-month U.S. Treasury bill rate, CPI	ADF tests with simulated critical values that adjust for moving-average components fail to reject a unit root in the nominal interest rate, inflation rate, and EPRR.
Mishkin (1992)	M: 1953–1990	U.S.	1- and 3-month Treasury bill rates, CPI	ADF tests with simulated critical values that adjust for moving-average components fail to reject a unit root in the nominal interest rate and inflation rate. AEG tests typically reject the null of no cointegration, indicating a stationary EPRR.
Wallace and Warner (1993)	Q: 1948–1990	U.S.	3-month Treasury bill rate, 10-year government bond yield, CPI	ADF tests fail to reject a unit root in the long-term nominal interest rate and inflation rate. Johansen (1991) procedure provides evidence that the variables are cointegrated and that the EPRR is stationary.
Engsted (1995)	Q: 1962–1993	13 OECD countries	Long-term bond yield, CPI	ADF tests fail to reject a unit root in nominal interest rates and inflation rates, while cointegration tests present ambiguous results on the stationarity of the EPRR across countries.
Mishkin and Simon (1995)	Q: 1962–1993	Australia	13-week government bond yield, CPI	ADF tests fail to reject a unit root in the nominal interest rate and inflation rate. AEG tests typically fail to reject the null hypothesis of no cointegration, indicating a nonstationary EPRR.
Crowder and Hoffman (1996)	Q: 1952–1991	U.S.	3-month Treasury bill rate, implicit consumption deflator, Livingston inflation expectations survey, tax data from various sources	ADF test fails to reject a unit root in the nominal interest rate and inflation rate after accounting for moving-average components. Johansen (1991) procedure rejects the null of no cointegration and supports a stationary EPRR.



Table 1 (continued)

Study	Sample	Countries	Nominal interest rate and price data	Results on the long-run properties of nominal interest rates, inflation rates, and real interest rates
Kousta and Serletis (1999)	Q: Data begin from 1957–1972; all data end in 1995	11 OECD countries	Various short term nominal interest rates, CPI	ADF tests usually fail to reject a unit root in nominal interest rates and inflation rates, while KPSS tests typically reject the null of stationarity, indicating nonstationary nominal interest rates and inflation rates. AEG tests typically fail to reject the null of no cointegration, indicating a nonstationary EPRR.
Bierens (2000)	M: 1954–1994	U.S.	Federal funds rate, CPI	New test provides evidence of nonlinear cotrending between the nominal interest rate and inflation rate, indicating a stationary EPRR. New test, however, cannot distinguish between nonlinear cotrending and linear cointegration.
Rapach (2003)	A: Data begin in 1949–1965; end in 1994–1996	14 industrialized countries	Long-term government bond yield, implicit GDP deflator	ADF tests fail to reject a unit root in all nominal interest rates and in 13 of 17 inflation rates. This indicates a nonstationary EPRR for the four countries with a stationary inflation rate. AEG tests typically fail to reject a unit root in the EPRR for the 13 countries with a nonstationary inflation rate, indicating a nonstationary EPRR for these countries.
Rapach and Weber (2004)	Q: 1957–2000	16 OECD countries	Long-term government bond yield, CPI	Ng and Perron (2001) unit root tests typically fail to reject a unit root in nominal interest rates and inflation rates. Ng and Perron (2001) and Perron and Rodriguez (2001) tests usually fail to reject the null of no cointegration, indicating a nonstationary EPRR in most countries.
Karanasos et al. (2006)	A: 1876–2000	U.S.	Long-term government bond yield, CPI	95% confidence interval for the EPRR's $\rho$ is [0.97, 0.99]. There is evidence of long-memory, mean-reverting behavior in the EPRR.
Lai (1997)	Q: 1974–2001	8 industrialized and 8 developing countries	1- to 12-month Treasury bill rates, CPI, DRI inflation forecasts	ADF and KPSS tests indicate a unit root in the nominal interest rate, inflation rate, and expected inflation rate. There is evidence of long memory, mean-reverting behavior in the EARR and EPRR.
Tsay (2000)	M: 1953–1990	U.S.	1- and 3-month Treasury bill rates, CPI	There is evidence of long-memory, mean-reverting behavior in the EPRR.

Table 1 (continued)

Study	Sample	Country	Nominal interest rate and price data	Results on the properties of nominal interest rates, inflation rates, and real interest rates
Sun and Phillips (2004)	Q: 1934–1994	U.S.	3-month Treasury bill, inflation forecasts from the <i>Survey of Professional Forecasters</i> , CPI	Bivariate exact Whittle estimator indicates long-memory behavior in the EARR. There is no evidence of a fractional cointegrating relationship between the nominal interest rate and expected inflation rate.
Pipatchaipoom et al. (2005)	M: 1971–2003	U.S.	Eurodollar rate, CPI	Exact Whittle estimator provides evidence of long memory, mean-reverting behavior in the EARR.
Maki (2003)	M: 1972–2000	Japan	10-year bond rate, call rate, CPI	Breitung (2002) nonparametric test that allows for nonlinear short-run dynamics provides evidence of cointegration between the nominal interest rate and inflation rate; cointegrating vector is not estimated, however, so it is not known if the cointegrating relationship is consistent with a stationary EPRR.
Million (2004)	M: 1951–1999	U.S.	3-month Treasury bill rate, CPI	Luukkonen et al. (1988) test rejects linear short-run dynamics for the adjustment to the long-run equilibrium EPRR. A smooth transition autoregressive (ESTAR) model exhibits asymmetric mean reversion in the EPRR, depending on the level of the EPRR.
Christopoulos and Leòn-Ledesma (2007)	Q: 1960–2004	U.S.	3-month Treasury bill rate, CPI	Choi and Saikkonen (2005) test provides evidence of nonlinear cointegration between the nominal interest rate and inflation rate. Exponential smooth transition regression (ESTR) model fits best over the full sample and the first subsample (1960–1978), while a logistic smooth transition regression (LSTR) model fits best over the second subsample (1979–2004). Estimated ESTR model for 1960–1978 is not consistent with a stationary real EPRR for any inflation rate, and estimated LSTR model for 1979–2004 is consistent with a stationary EPRR only when the inflation rate is above approximately 3%.
Kousta and Lamarche (2008)	A: 1960–2004	G-7 countries	3-month government bill rates, CPI	ADF and KPSS tests provide evidence of a unit root in the nominal interest rate and inflation rate. Bec et al. (2004) nonlinear unit root and Hansen (1996, 1997) linearity tests indicate that the EPRR can be suitably modeled as a three-regime self-exciting autoregressive (SETAR) process in Canada, France, and Italy.

Table 1 (continued)

Study	Sample	Country	Nominal interest rate and price data	Results on the properties of nominal interest rates, inflation rates, and real interest rates
Garcia and Perron (1996)	Q: 1961–1986	U.S.	3-month Treasury bill rate, CPI	An estimated autoregressive model with a three-state Markov-switching process for the mean indicates that the EPRR was in a “moderate”-mean regime for 1961–1973, a “low”-mean regime for 1973–1980, and a “high”-mean regime for 1980–1986. EPRR is stationary with little persistence within these regimes.
Clemente et al. (1998)	Q: 1980–1995	U.S., U.K.	Long-term government bond yield, CPI	ADF tests that allow for two structural breaks in the mean reject a unit root in the EPRR, indicating that the EPRR is stationary within regimes defined by structural breaks.
Caporale and Grier (2000)	Q: 1961–1986	U.S.	3-month Treasury bill rate, CPI	Bai and Perron (1998) methodology provides evidence of multiple structural breaks in the mean EPRR.
Bai and Perron (2003)	Q: 1961–1986	U.S.	3-month Treasury bill rate, CPI	Bai and Perron (1998) methodology provides evidence of multiple structural breaks in the mean EPRR.
Lai (2004)	M: 1978–2002	U.S.	1-year Treasury bill rates, inflation expectations from the Univ. of Michigan <i>Survey of Consumers</i> , CPI, federal marginal income tax rates for 4-person families	ADF tests allowing for a structural break in the mean reject a unit root in the tax-adjusted or unadjusted EARR, indicating that the EARR is stationary within regimes defined by the structural break.
Rapach and Wohar (2005)	Q: 1960–1998	13 OECD countries	Long-term government bonds yield, CPI, marginal tax rate data (Padovano and Galli, 2001)	The Bai and Perron methodology (1998) provides evidence of structural breaks (usually multiple) in the mean EPRR and mean inflation rate for all 13 countries.
Lai (2007)	Q: 1974–2001	8 industrialized and 8 developing countries	1- to 12-month Treasury bill rates, deposit rates, CPI	ADF tests allowing for a structural break in the mean reject a unit root in the EPRR for most countries, indicating that the EPRR is stationary within regimes defined by the structural break.

Note: A, Q, M indicate annual, quarterly, and monthly data frequencies.

Table 2: Unit root test statistics, U.S. data, 1953:1–2007:2

Variable	ADF	$MZ_{\alpha}$
Three-month Treasury bill rate	−2.49 [7]	−4.39 [8]
PCE deflator inflation rate	−2.72 <sup>†</sup> [4]	−5.20 [5]
<i>Ex post</i> real interest rate	−3.06* [6]	−18.83** [2]
Per capita consumption growth	−4.99** [4]	−42.07** [2]

Notes: The ADF and  $MZ_{\alpha}$  statistics correspond to a one-sided (lower-tail) test of the null hypothesis that the variable has a unit root against the alternative hypothesis that the variable is stationary. The 10%, 5%, and 1% critical values for the ADF statistic are −2.58, −2.89, and −3.51; the 10%, 5%, and 1% critical values for the  $MZ_{\alpha}$  statistic are −5.70, −8.10, and −13.80. The lag order for the regression model used to compute the test statistic is reported in brackets. <sup>†</sup>, \*, and \*\* indicate significance at the 10%, 5%, and 1% levels.

Table 3: Cointegration test statistics and cointegrating coefficient estimates, U.S. three-month Treasury bill rate and inflation rate, 1953:1–2007:2

<u>A. Cointegration tests</u>		
AEG	$MZ_{\alpha}$	Trace
$-3.07^{\dagger}$ [6]	$-17.11^*$ [2]	$19.96^{\dagger}$ [4]
<u>B. Coefficient estimates</u>		
Estimation method	$\theta_0$	$\theta_1$
Dynamic OLS	$2.16^*$ (1.01)	$0.86^{**}$ (0.24)
Johansen (1991) maximum-likelihood	$0.39$ (1.21)	$1.44^{**}$ (0.29)

Notes: The AEG and  $MZ_{\alpha}$  statistics correspond to a one-sided (lower-tail) test of the null hypothesis that the three-month Treasury bill rate and inflation rate are not cointegrated against the alternative hypothesis that the variables are cointegrated. The 10%, 5%, and 1% critical values for the AEG statistic are  $-3.07$ ,  $-3.37$ , and  $-3.96$ ; the 10%, 5%, and 1% critical values for the  $MZ_{\alpha}$  statistic are  $-12.80$ ,  $-15.84$ , and  $-22.84$ . The trace statistic corresponds to a one-sided (upper-tail) test of the null hypothesis that the three-month Treasury bill rate and inflation rate are not cointegrated against the alternative hypothesis that the variables are cointegrated. The 10%, 5%, and 1% critical values for the trace statistic are  $18.47$ ,  $20.66$ , and  $24.18$ . The lag order for the regression model used to compute the test statistic is reported in brackets.  $^{\dagger}$ ,  $*$ , and  $^{**}$  indicate significance at the 10%, 5%, and 1% levels. Standard errors are reported in parentheses.

Table 4: Bai and Perron (1988) test statistics and estimation results for the U.S. *ex post* real interest rate, 1953:1–2007:2

Test statistic		Regime	Estimated <i>ex post</i> real interest rate mean
$UD_{max}$	14.84**	1953:1–1972:3 [1969:2, 1973:4]	1.22** (0.17)
$WD_{max}$ (5%)	27.06*	1972:4–1980:3 [1979:1, 1980:4]	–0.55 (0.38)
$F(1 0)$	12.92**	1980:4–1989:3 [1984:3–1994:2]	4.58** (0.71)
$F(2 1)$	17.89**	1989:4–2007:2	1.82** (0.52)
$F(3 2)$	17.89**		
$F(4 3)$	10.37 <sup>†</sup>		
$F(5 4)$	10.37		

Notes: The  $UD_{max}$  statistic corresponds to a one-sided (upper-tail) test of the null hypothesis of 0 breaks against the alternative hypothesis of an unknown number of breaks given an upper bound of 5; 10%, 5%, and 1% critical values are 7.46, 8.88, and 12.37. The  $WD_{max}$  (5%) statistic corresponds to a one-sided (upper-tail) test of the null hypothesis of 0 breaks against the alternative hypothesis of an unknown number of breaks given an upper bound of 5; critical value is 9.91. The  $F(l+1|l)$  statistics correspond to a one-sided (upper-tail) test of the null hypothesis of  $l$  breaks against the alternative hypothesis of  $l+1$  breaks. 10%, 5%, and 1% critical values are  $F(1|0)$ , 7.04, 8.58, and 12.29;  $F(2|1)$ , 8.51, 10.13, and 13.89;  $F(3|2)$ , 9.41, 11.14, and 14.80;  $F(4|3)$ , 10.04, 11.83, and 15.28;  $F(5|4)$ , 10.58, 12.25, and 15.76. Break dates defining the regimes are estimated using the Bai and Perron (1998) methodology; 90% confidence intervals for the break dates defining the regimes are reported in brackets. Standard errors are reported in parentheses. <sup>†</sup>, \*, and \*\* indicate significance at the 10%, 5%, and 1% levels.

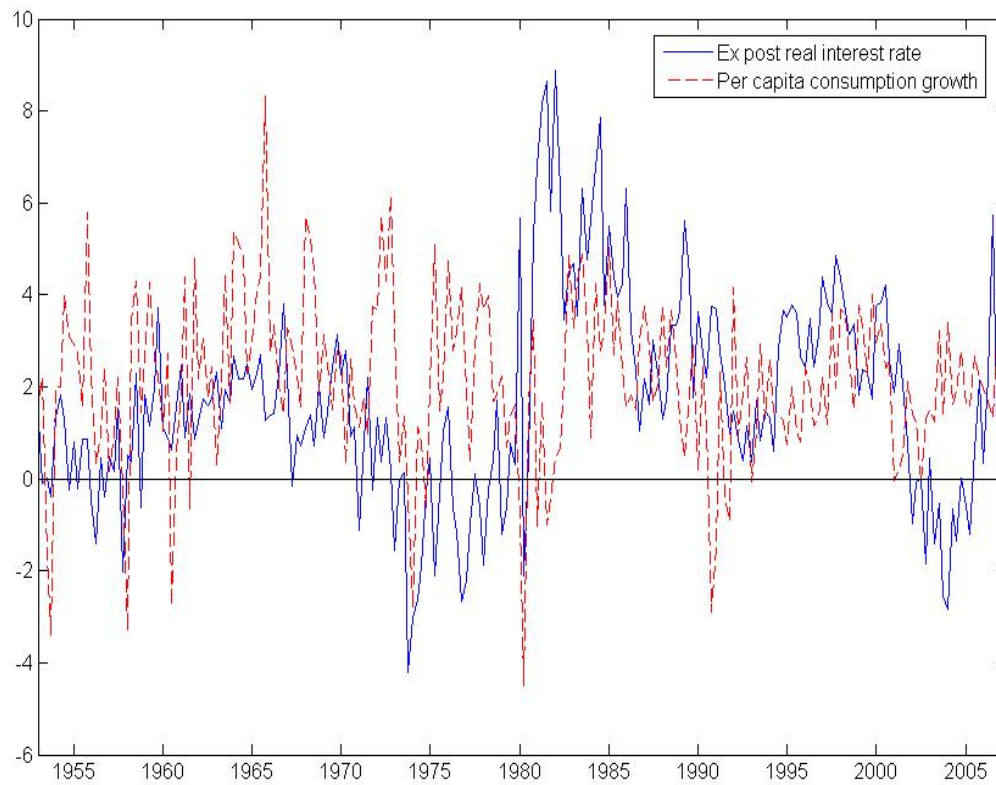


Figure 1: U.S. *ex post* real interest rate and real per capita consumption growth, 1953:1–2007:2

Notes: Figure plots the U.S. *ex post* three-month real interest rate and annualized per capita consumption growth. Consumption is measured as the sum of nondurable goods and services consumption.

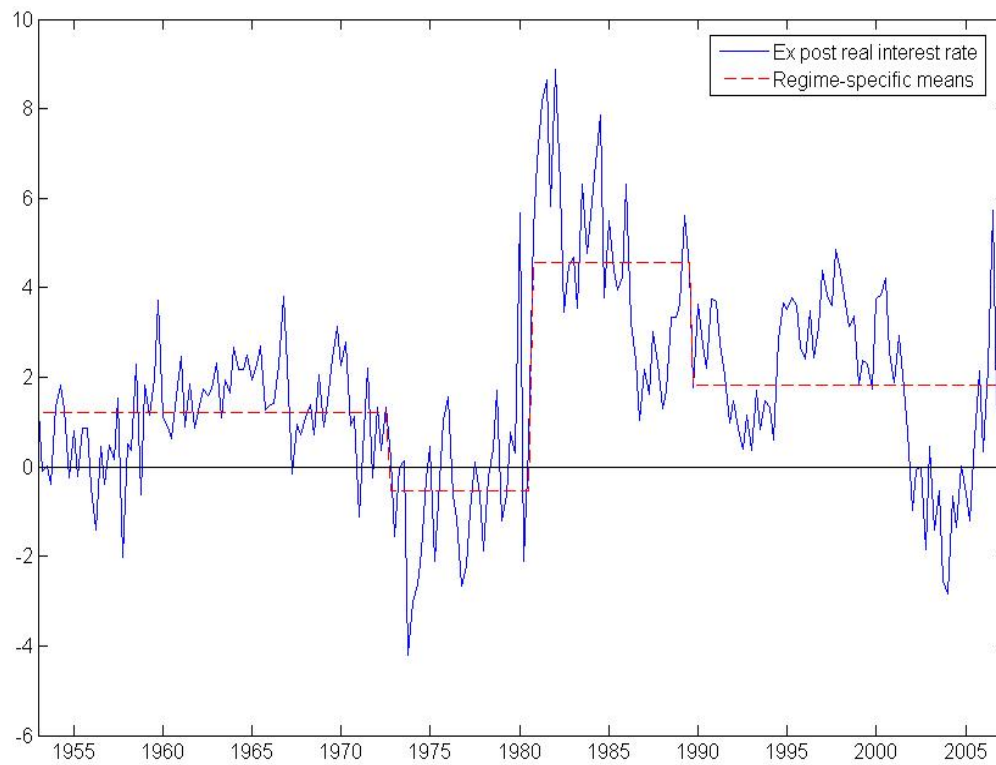


Figure 2: U.S. *ex post* real interest rate and regime-specific means, 1953:1–2007:2

Note: Figure plots the U.S. *ex post* real interest rate and means for the regimes defined by the structural breaks estimated using the Bai and Perron (1998) methodology.



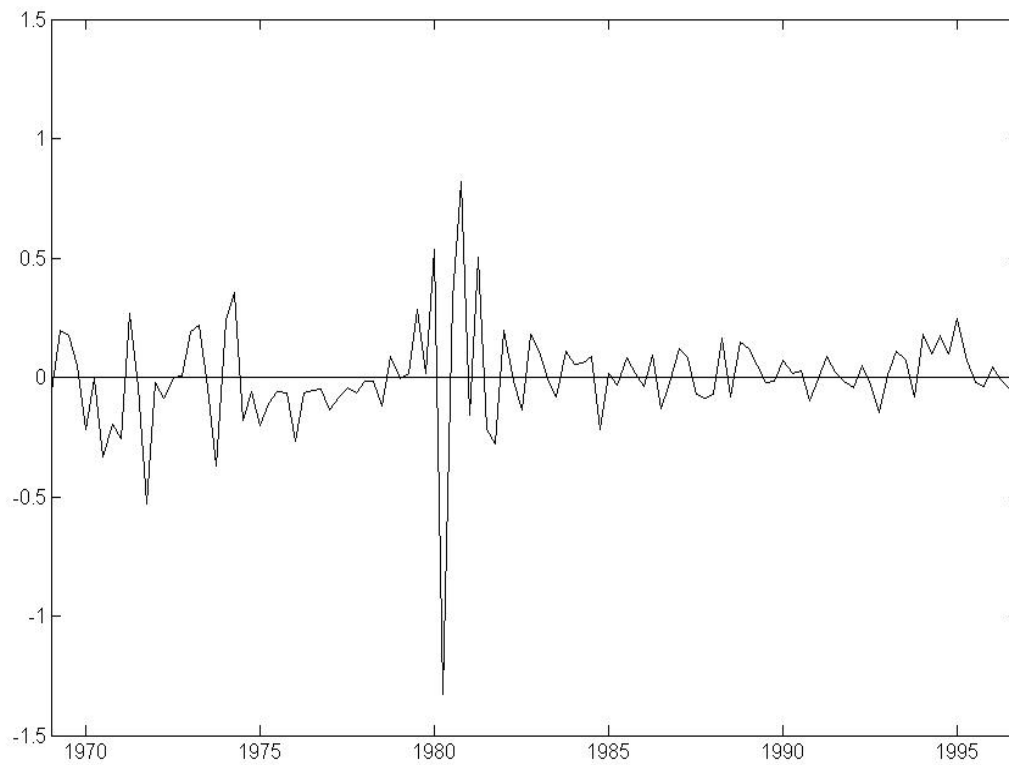


Figure 3: Romer and Romer (2004) measure of monetary policy shocks, 1969:1–1996:4

Note: A positive (negative) value corresponds to a contractionary (expansionary) monetary policy shock.

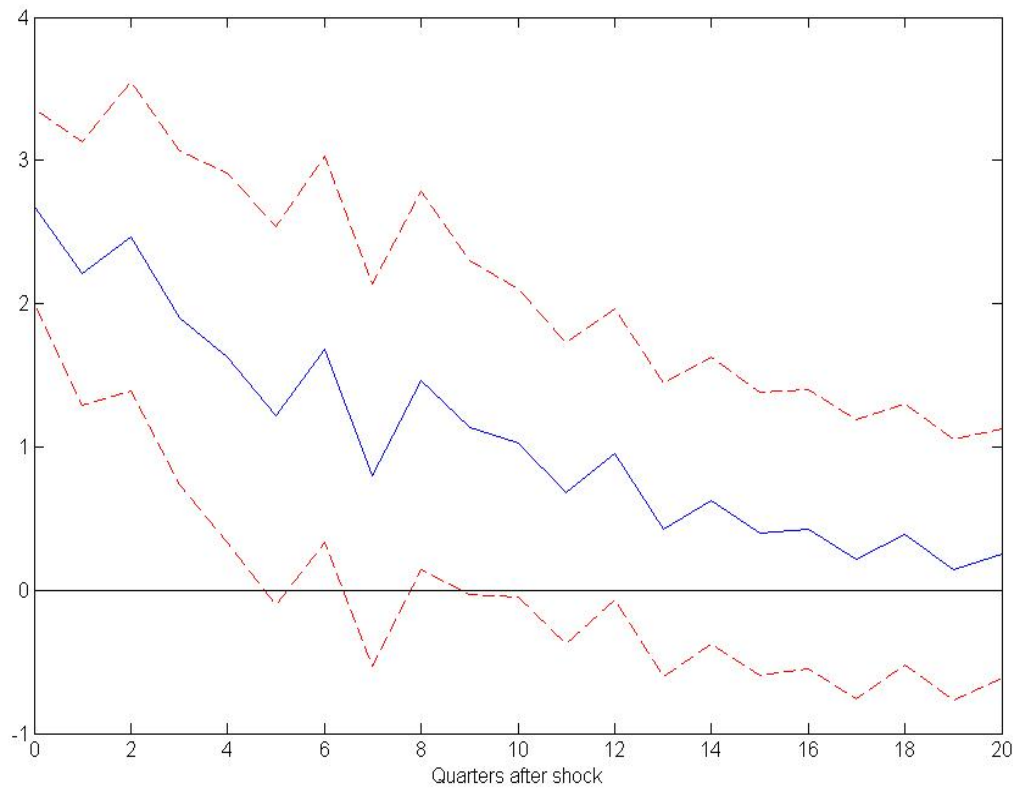


Figure 4: U.S. *ex post* real interest rate response to a contractionary Romer and Romer (2004) monetary policy shock

Notes: The response is based on an autoregressive distributed lag model estimated for 1969:1–1996:4. Dashed lines delineate two-standard-error bands. The response is to a shock of size 0.5.